Solutions to the Midterm: Astro 242

1. Answer the following questions without using any equations or symbols. Be brief and direct.

(a) Explain the evidence for dark matter in spiral galaxies. Circular velocities of stars are constant out to radii much larger than the visible parts of the disk. More generally,the gravitational field due to the observed disk does not produce the observed velocities of stars in the disk.

(b) In what direction does the force of dynamical friction act? How does it depend on the mass of the moving blob? Explain this dependence. The force acts opposite to the direction in which the blob is traveling. It depends on the square of the blob's mass. The reason for the two powers of mass is that a wake is set up trailing the blob, the strength of which is proportional to the mass. The gravitational pull of this wake – i.e. dynamical friction – is also proportional to the mass.

(c) Are the stars in spiral arms typically younger or older than stars in the rest of the disk? Why? Stars in the spiral arms are younger. Spiral arms correspond to overdense regions. These are more likely to condense to form stars. However, a given star is not fixed in the arm: it moves through it, so the young stars are produced in the arms but then when they move out, the older ones remain in the disk.

2. What is the circular velocity of a star moving in a spherical halo of dark matter with a Navarro,Frenk, and White profile:

$$\rho = \frac{r_s}{r} \frac{\rho_0}{(1 + r/r_s)^2}.$$

The mass enclosed within a radius r is

$$M(r) = 4\pi \int_0^r dr' r'^2 \rho(r').$$

Define $x \equiv r'/r_s$. Then the integral is

$$M(r) = 4\pi \rho_0 r_s^3 \int_0^{r/r_s} dx \frac{x}{(1+x)^2}.$$

Integrate by parts to get

$$M(r) = 4\pi\rho_0 r_s^3 \left[\frac{-x}{1+x}\Big|_0^{r/r_s} + \int_0^{r/r_s} \frac{dx}{1+x} \right].$$

The second integral is simply $\ln(1+x)$, so

$$M(r) = 4\pi\rho_0 r_s^3 \left[\frac{-r}{r_s + r} + \ln(1 + r/r_s) \right].$$

The circular velocity induced by this mass is $GM(r)/r$, so

$$v^2 = 4\pi G\rho_0 r_s^3 \left[\frac{-1}{r_s + r} + \frac{\ln(1 + r/r_s)}{r} \right].$$

3. What is the maximum mass globular cluster you expect to find within the central 4 kpc of the the Andromeda Galaxy? Use the fact that the circular velocity of stars in Andromeda is roughly constant 250 km sec^{-1}.

All satellites more massive than $M_{\rm max}$ have fallen in to the center because of dynamical friction. Inverting a formula we derived in class

$$M_{\rm max} = \frac{r^2 v_c}{2\beta G t}.$$

Here r^2 and v_c were given in the problem, t is the age of the galaxy which you can take to be roughly ten billion years. And 2β is roughly $4\pi \ln\Lambda \sim 12\pi$. Therefore,

$$M_{\rm max} = \frac{(4 \times 3 \times 10^{21}{\rm cm})^2 (2.5 \times 10^7 {\rm cm\ sec}^{-1}}{12\pi 6.67 \times 10^{-8} {\rm cm}^3 {\rm sec}^{-2} {\rm g}{-1}(10^{10} \times 3 \times 10^7 {\rm sec}}.$$

Plugging in numbers leads to

$$M_{\rm max} = 4.8 \times 10^{39}{\rm g} = 2.4 \times 10^6 M_\odot.$$

4. A Cepheid variable was discovered in a galaxy called NGC 1365. The Cepheid had a period of 35 days. What is its absolute magnitude? Its observed magnitude is 26.81; how far away is it? The velocity of the NGC 1365 was 1652 km sec^{-1}. What is the inferred Hubble constant?

The absolute magnitude is $M = -2.8 \log(P/\text{days}) - 1.43$. So $M = -5.75$. Its distance is determined via $m - M = 5 \log(d/10\text{pc})$, so $d = 10\text{pc}10^{(26.81+5.75)/5} = 33$ Mpc. The Hubble constant is $v/d = 51$ km sec^{-1} Mpc^{-1}.

5. What is the limit of $I_0(y)K_0(y) - I_1(y)K_1(y)$ as $y \to \infty$? Hint: You do not need to know anything about Bessel functions to do this problem.

We recognize this Bessel function combination as $v^2 h_R/(\pi G \Sigma_0 R^2)$. We know that at very large distances, the whole disk will look like a point, so that $v^2 = GM/R$ where M is the total mass of the disk. The total mass of the disk though is Σ_0 times the area of the disk: $M = \Sigma_0 \pi h_R^2$. So,

$$I_0(y)K_0(y) - I_1(y)K_1(y) \to (GM/R)h_R/(\pi G \Sigma_0 R^2) = (h_R/R)^3 = (1/2y)^3$$

since $y \equiv R/2h_R$.